

Linear superpositions of gap solitons in periodic Kerr media

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The existence of a novel type of solitons in periodic Kerr media constructed as superposition of noninteracting gap-solitons of different kinds (bright, dark and periodic) is first demonstrated. The periodic modulation of the nonlinearity is used to suppress the cross phase modulation between components to allow the superimposed beam to propagate for long distances. © 2011 Optical Society of America

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Photonic structures with periodic modulations of parameters are ideal systems to investigate nonlinear properties of wave dynamics (see e.g. [1]). In this context, arrays of optical waveguides are one of the most used configurations for practical implementations of various physical phenomena involving optical solitons. To avoid problems of cross-phase modulations affecting beams consisting of different modes, which differ by frequencies and/or by wavelengths, the nonlinear properties of the light propagation in these structures have been investigated mainly for monochromatic light. Instability induced by cross-phase modulation, however, could arise also for monochromatic light, if linear superposition of nonlinear modes of different wavelengths are considered. Meantime, stabilizing effects on the dynamics achieved by means of periodic modulations of the nonlinearity have been explored in the context of Bose-Einstein condensates in optical lattices where it has been shown that periodic modulations of the interatomic interaction allow one to induce Bloch oscillations [2], intra-band (Rabi) oscillations [3] and dynamical localization [4] of gap solitons which persist for long times (long propagation distances in the optical context). In view of the analogies between photonic lattices and Bose-Einstein condensates in optical lattices it is of interest to investigate the possibility of using periodic modulations of the nonlinearity as a tool for controlling the cross phase modulation in superimposed beams.

The aim of this Letter is to demonstrate the existence of a novel type of solitons in periodic Kerr media made as superpositions of noninteracting gap-solitons of different wavevectors and different kinds (bright, dark and periodic). The periodic modulation of the nonlinearity is used to suppress the cross phase modulation between components to allow the superimposed beam to propagate for long distances.

As model equation for the propagation of a monochromatic beam along z -axis in a periodic Kerr media we use the following nonlinear Schrödinger equation for the di-

mensionless electric field envelope q (see e.g. [1]):

$$iq_z = -q_{xx} + V(x)q + G(x)|q|^2q. \quad (1)$$

Here $V(x) = V(x + \pi)$ and $G(x) = G(x + \pi)$ denoting the linear refractive index and the nonlinearity modulations in the transverse direction, respectively. Hereafter we fix $V(x) = V \cos(2x)$ and $G(x) = \sigma + G \cos(2x)$, both taken of period π without loss of generality. The average Kerr index σ , as well as amplitudes of linear refractive index, V , and of nonlinearity, G , modulations are considered as free parameters, used below for achieving zero cross phase modulation conditions. The periodicity of the linear refractive index induces a band structure determined by the eigenvalue problem $d^2\varphi_{\alpha,k}/dx^2 - V(x)\varphi_{\alpha,k} = b_{\alpha}(k)\varphi_{\alpha,k}$, where the eigenvalues $b_{\alpha}(k)$ (propagation constants) are periodic functions of the Bloch vector $k \in [-1, 1]$ and the associated orthonormal set of eigenmodes $\varphi_{\alpha,k}$ satisfy the Bloch condition $\varphi_{\alpha,k}(x + \pi) = e^{ik\pi}\varphi_{\alpha,k}(x)$ (here α denotes the band index). We are particularly interested in the modes bordering different edges of the same or of different bands. Respectively, we use subscripts l, u to denote lower and upper propagation constant values ($b_l < b_u$).

The self-phase modulation of small amplitude solitons bifurcating from band edges $b_{l,u}$ is described by $\chi_{l,u} = \int_0^\pi G(x)|\varphi_{l,u}|^4 dx$, while the cross-phase modulation is determined by $\chi = \int_0^\pi G(x)|\varphi_l|^2|\varphi_u|^2 dx$. Then necessary conditions for the existence of the bright solitons, combined with the requirement for zero cross-phase modulation, allowing them to bifurcate independently from different edges, take the form

$$D_l\chi_l < 0, \quad D_u\chi_u < 0, \quad \chi = 0, \quad (2)$$

where $D_{l,u}$ denote the linear diffraction coefficients of the respective l, u modes: $D_{l,u} = -d^2b_{l,u}/dk^2$ (see [2] for details). These conditions can be satisfied for proper choices of $V(x)$ and $G(x)$.

Let us now consider a superposition of two localized solutions, $w_{1,2}(x)e^{ib_{1,2}z}$ of Eq. (1), i.e.

$$q(x, z) = w_1(x)e^{ib_1z} + w_2(x)e^{ib_2z+i\Theta}. \quad (3)$$

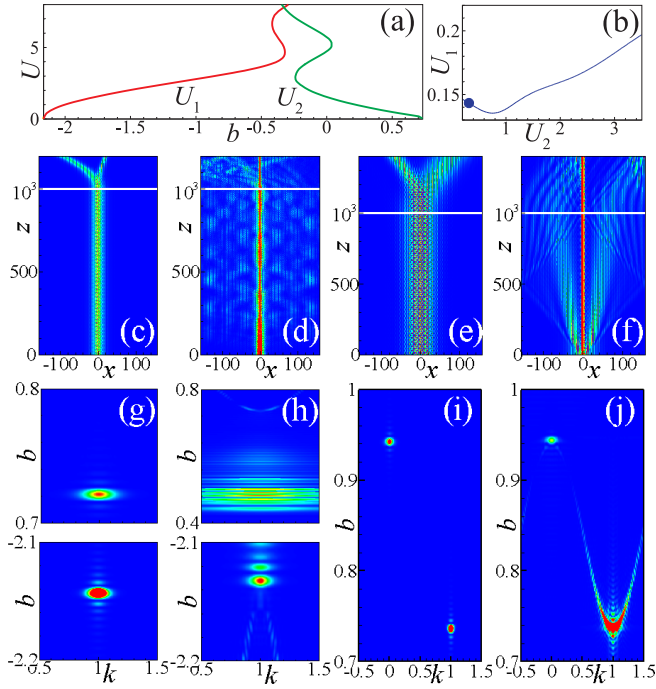


Fig. 1. (color on line) (a) Families of the modes bifurcating from the lower, U_1 , and upper U_2 edges of the first gap: $b \in [-2.1659, 0.7332]$ (the edges coincide with the panel boundaries); (b) U_1 vs U_2 for the solitons fulfilling (4); (c-f) Intensity distributions $|q(x, z)|^2$ and (g-j) spectra $|Q(b, k)|^2$ at distance $z_0 = 1000$ of input beams (3) consisting of two superimposed solitons with parameters: (c,g) $U_1 = 0.32$ ($b_1 = -2.15$), $U_2 = 0.143$ ($b_2 = 0.71$); (d,h) $U_1 = 0.645$ ($b_1 = -2.1$), $U_2 = 0.33$ ($b_2 = 0.6$); (e,i) $U_1 = 2.77$ ($b_1 = 0.732$), $U_2 = 1.7$ ($b_2 = 0.938$); (f,j) $U_1 = 2.77$ ($b_1 = 0.732$), $U_2 = 5.28$ ($b_2 = 1.0$). In panels (c,d,g,h) solitons are near edges of the first gap while in panels (e,f,i,j) near edges of the first band. Other parameters are $\sigma = -1$, $V = -3$, $G = 3.05$ (a-d,g,h), $G = 1.282$ (e,f,i,j).

Here without loss of generality the functions $w_j(x)$ ($j = 1, 2$) are taken to be real solutions of the stationary equation $d^2 w_j / dx^2 + b_j w_j - V(x) w_j + G(x) w_j^3 = 0$ while the constant Θ denotes a relative phase. For propagation constants sufficiently close to the gap edges, i.e. for $b_1 \approx b_l$ and $b_1 \approx b_u$, these solutions can be represented as $w_{1,2}(x) = A_{1,2}(\mu_{1,2} x) \varphi_{l,u}(x)$, where $\mu_{1,2} = \sqrt{|b_{1,2} - b_{l,u}|} \ll 1$ is a small parameter characterizing detunings to the gap, and $A_j(\cdot)$ are slowly varying amplitudes. Then (2) ensures that $\chi \sim \mu_j^2 \ll \chi_{l,u}$ so that the cross phase modulation of Bloch states implies strong suppression of the cross-phase modulation of the small amplitude solitons $w_{1,2}(x)$. Respectively, we expect the superposition of such solutions to approach a stable solutions of the nonlinear equation. When the detuning towards the gap increases, the condition (2) must be

modified, to ensure

$$\int_{-\infty}^{\infty} w_1 w_2 dx = 0, \quad \text{and} \quad \int_{-\infty}^{\infty} G(x) w_1^2 w_2^2 dx = 0. \quad (4)$$

To show that when (4) are satisfied the superposition (3) becomes highly accurate approximation of the exact solutions we recall that Eq. (1) admits two conserved quantities: the energy flow $U[q] = \int_{-\infty}^{\infty} |q|^2 dx$ and the Hamiltonian $H[q] = \int_{-\infty}^{\infty} [|q_x|^2 + V(x)|q|^2 + \frac{1}{2} G(x)|q|^4] dx$. For the solutions (4), then the above conditions imply: $U[q] = U[w_1] + U[w_2]$ and $H[q] = H[w_1] + H[w_2]$ so that, in spite of the nonlinearity in Eq. (1), the superimposed beam (3) will propagate as an exact solution, without appreciable distortions. In the case of solitons with $b_{1,2}$ bifurcating from opposite edges of the first gap [Fig. 1(a)], the first of conditions (4) is satisfied due to the parity of solutions $w_1(x)$ (odd) and $w_2(x)$ (even). As to the cross-phase modulation, for fixed values of V and G , for each even soliton with b_2 (and the energy flux $U_2 = U[w_2]$) there exists its unique odd counterpart with b_1 (and the energy flux $U_1 = U[w_1]$), satisfying the second condition in (4). We remark that for two low-amplitude solitons the second condition in (4) is usually satisfied for $G \gtrsim G_0$, where G_0 denotes the amplitude of $G(x)$ for which $\chi = 0$ (for the refractive index parameters of Fig.1(a) $G_0 \approx 2.98$ while $G = 3.05$). For this situation the respective dependence $U_1(U_2)$ is illustrated in Fig. 1(b). We however emphasize that only stable points of the presented curve can be used in an experiment. Typically, only small-amplitude solitons are stable: $U_1 \lesssim 0.5$ for the parameters chosen in Fig. 1.

The fact that the superposition can propagate without significant distortion is confirmed by Figs. 1(c,e). In order to emphasize the distortionless propagation of the beam, at $z_0 = 1000$ (denoted in Figs. 1(c-f) by horizontal lines) we applied a refractive index gradient in the transverse direction, modeled by adding the term $0.001 x q(x, z)$ in the right-hand side of Eq. (1) for $z > z_0$. We observe that when conditions (4) are met [Figs. 1(c,e)] the two superimposed solitons propagate undistorted until the refractive gradient is applied and after that it splits into two gap solitons propagating in opposite directions, due to the opposite signs of the diffraction coefficients $D_{l,u}$. If, however, Eq. (4) is strongly violated we get that the superimposed beam is destroyed after relatively short propagation distances (i.e. for $z \ll z_0$) [see Figs. 1(d,f)]. The spectra $Q(b, k) = \int_{-\infty}^{\infty} q(x, z) e^{-ibz} \varphi_{\alpha,k}(x) dz dx$ of the superimposed beams computed at $z = z_0$ are shown in Figs. 1(g-j) (each panel corresponding to the spatial dynamics shown in the middle row just above it). We see that when the zero cross phase modulation condition is satisfied [Figs. 1(g,i)] the output spectrum is centered around the same parameters b, k in the Fourier space as the respective input beam (in spite of the very long propagation distance), while in the opposite case, [Figs. 1(h,j)], the spectrum becomes strongly deformed. Note the differences between spectral

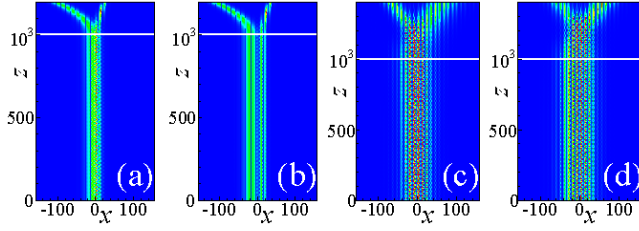


Fig. 2. (color on line) Propagation of two superimposed gap solitons with centers shifted at the input by $\Delta x = 4\pi$ (panels a,c), or by $\Delta x = 10\pi$ (panels b,d). Soliton parameters are the same as in Fig. 1(c) (panels a,b), or Fig.1(e) (panels c,d).

widths and transverse spatial extension of the superimposed beam in Figs. 1(c,g) and the ones in Figs. 1(e,i). While for the former case (component solitons at opposite edges of the first gap) the orthogonality condition is always satisfied due to the parity, in the latter case (component solitons at opposite edges of the first band) this is true only in the small amplitude limit, as we remarked before (in this case the orthogonality of the solitons can be achieved only through the orthogonality of the Bloch states). This explains the wider width in the transverse direction of the latter beam and the narrowing of the spectral widths of the component gaps-solitons.

In Fig. 2 we have shown the effect of an initial displacement Δx on the propagation of the two superimposed solitons in Fig. 1(c,e). We see that the superpositions remain stable both for small [Figs.2(a,c)] and large [Figs.2(b,d)] displacements. This is understood by the fact that, being the solitons of small amplitudes, nonlinear-orthogonality condition (4) is only slightly violated by small displacements.

The above results can be extended to superpositions of nonlinear periodic waves with a bright soliton [Fig. 3(a-c)] and to superpositions of dark solitons [Fig. 3(d-f)]. In the first case the periodic (even) wave $w_2(x)$ bifurcates from the lower edge of semi-infinite gap [Fig. 3(a)], while the bright (odd) soliton $w_1(x)$ bifurcates from the lower edge of first gap [Fig. 3(b)]. The both entities are stable; we however note the difference in their amplitudes [c.f. the scales of $|q|^2$ in Figs. 3(a) and 3(b)]. In the second example, the dark solitons bifurcate from the opposite edges of the first gap towards the first [Fig. 3(d)] and the second [Fig. 3(e)] allowed bands. In both cases the condition for zero cross phase modulation is the same as in (4) (or $\chi = 0$ in the small amplitude limit) but conditions for existence and stability of the modes must be changed as $D_l \chi_l < 0$, $D_u \chi_u > 0$ and $D_{l,u} \chi_{l,u} > 0$ for periodic wave-bright soliton and dark soliton - dark soliton superpositions, respectively. In the bottom panels of Fig. 3 are shown the output spectra computed at $z = 1000$ of the corresponding superimposed modes. In both cases we see that the linear superposition propagates in stable manner for a long distance as consequence of the

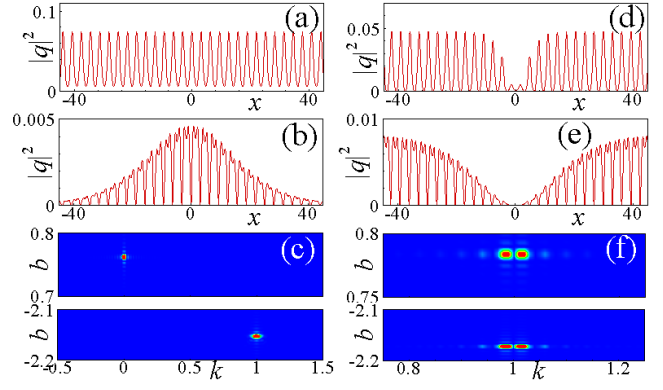


Fig. 3. (color on line) (a,b) Even periodic background with the energy flux per period $U_2 = \int_0^\pi w_2^2(x)dx = 0.103$ and $b_2 = 0.76$ (panel a), superimposed with an odd soliton having $U_1 = 0.132$ and $b_1 = -2.16$ (panel b) for the parameters $\sigma = -1$, $V = -3$, $G = 6$; (d,e) Superposition of two dark solitons bordering edges of the gap $b \in [-2.1659, 0.7332]$: even dark soliton with $b_2 = 0.78$ (panel d) and odd dark soliton with $b_1 = -2.18$ (panel e) for $V = 3$, $\sigma = 1$, $G = 2.95$; (c,f) Spectrum $|Q(b, k)|^2$ relative to superpositions of the waves in (a,b) and in (d,e) respectively, both taken at $z_0 = 1000$.

zero cross phase modulation between beam components. Similar results can be derived for solitons subjected to refractive indexes and nonlinearity modulations of different periods, as the ones considered in Ref. [5].

In conclusion, we have demonstrated the possibility of stable propagation of new types of superimposed beams in periodic Kerr media which are constructed as linear superpositions of gap solitons of several types and different wavelengths. The Kerr nonlinearity was designed in such a manner that the cross-phase modulation between beam components is eliminated or strongly reduced. These results can be of practical interest for multi-component parallel transmission and manipulation of optical signals in nonlinear media.

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